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QCD power corrections from a simple model for the running coupling

Bryan R. Webber*

Cavendish Laboratory, University of Cambridge
Madingley Road, Cambridge CB3 0HE, U.K.
E-mail: webber@hep.phy.cam.ac.uk

Abstract: A simple parametrization of the QCD running coupling at low scales is introduced and used to illustrate various schemes for the estimation of non-perturbative power corrections. The 'infrared matching' scheme proposed earlier gives satisfactory results when combined with next-to-leading (or higher) order perturbative predictions. Estimates based on renormalons are shown to be inconsistent with universal behaviour of the running coupling.

Keywords: QCD, Phenomenological Models, Nonperturbative Effects.

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Contents

1. Introduction

In several recent papers [1]–[6] the hypothesis of a universal low-energy behaviour of the QCD running coupling has been used to estimate power-behaved corrections to various QCD observables. While the only essential parameters are a few moments of the coupling in the infrared region, the absence of a satisfactory explicit model for the non-perturbative contributions has made the arguments hard to follow and the approximation scheme unclear. In the present paper, a simple model is introduced with sufficiently good properties to allow quantitative comparisons of various proposed schemes for the estimation of power corrections.

Let us start by recalling that the running coupling has the general form

$$\alpha_s(k^2) \equiv \frac{1}{b_0} a(k^2/\Lambda^2)\,, \tag{1.1}$$

where for N_f active flavours

$$b_0 = \frac{33 - 2N_f}{12\pi} \tag{1.2}$$

and to 1-loop order at high energies

$$a(x) \sim \frac{1}{\ln x}\,. \tag{1.3}$$

A fundamental problem with simply continuing this expression to low energies is the Landau pole at $k^2 = \Lambda^2$ ($x = 1$). On general causality grounds, at least if the running coupling is to be related analytically to some physical observable, we expect singularities of the function $\alpha_s(k^2)$ to occur on the negative real axis only.

A number of explicit models have been proposed [7]–[10] for including non-perturbative contributions at low k^2 and cancelling the Landau pole. A particularly simple one with many good features is the 'analytic model' [8, 9],

$$a(x) = \frac{1}{\ln x} + \frac{1}{1-x} \, . \tag{1.4}$$

This expression has no Landau pole, only a branch point at $k^2 = 0$. However, the extra term introduces a $1/k^2$ correction to the logarithmic running of the coupling at large k^2. This would have to be cancelled somehow in quantities that are proportional to $\alpha_s(Q^2)$ in leading order but are not expected to have $1/Q^2$ corrections, such as the e^+e^- total cross section. In addition the numerical value of α_s becomes rather large, $\alpha_s(0) = 1/b_0 = 1.4$ for $N_f = 3$, which calls into question the notion of an expansion in powers of α_s at low scales. One might also prefer to have an expression with free parameters, other than the overall momentum scale Λ, that could be fitted to experimental data. The expressions suggested in ref. [7] do have free parameters, but they also have non-analytic terms, unwanted power corrections and/or singularities at unphysical values of k^2.

2. Model for the running coupling

If we require

- No power corrections larger than $1/k^{2p}$,
- Singularities on the negative real k^2 axis only,
- Some freedom to adjust the form and value at low k^2,

then the following generalization of eq. (1.4) suggests itself:

$$a(x) = \frac{1}{\ln x} + \frac{x+b}{(1-x)(1+b)} \left(\frac{1+c}{x+c}\right)^p \, . \tag{2.1}$$

For definiteness we shall consider here $b = 1/4$, $c = 4$, $p = 4$, i.e.

$$a(x) = \frac{1}{\ln x} + 125 \frac{1+4x}{(1-x)(4+x)^4} \, , \tag{2.2}$$

which leads to the form of α_s shown in fig. 1, where the values $N_f = 3$, $\Lambda = 0.25$ GeV have been chosen, giving $\alpha_s(Q^2) = 0.118$ at $Q = 91$ GeV. For this choice of parameters, $\alpha_s(k^2)$ lies very close to the perturbative expression (dot-dashed) above $k = 1$ GeV, the difference falling like $1/k^8$. Below 1 GeV, however, it remains finite and positive, reaching a maximum value of 0.8 at $k = 0.4$ GeV. As discussed below, the moments of this function are consistent with those deduced from data on power corrections. The multipole on the negative real axis looks somewhat unphysical, but should be regarded only as an approximation to a more complicated singularity structure.

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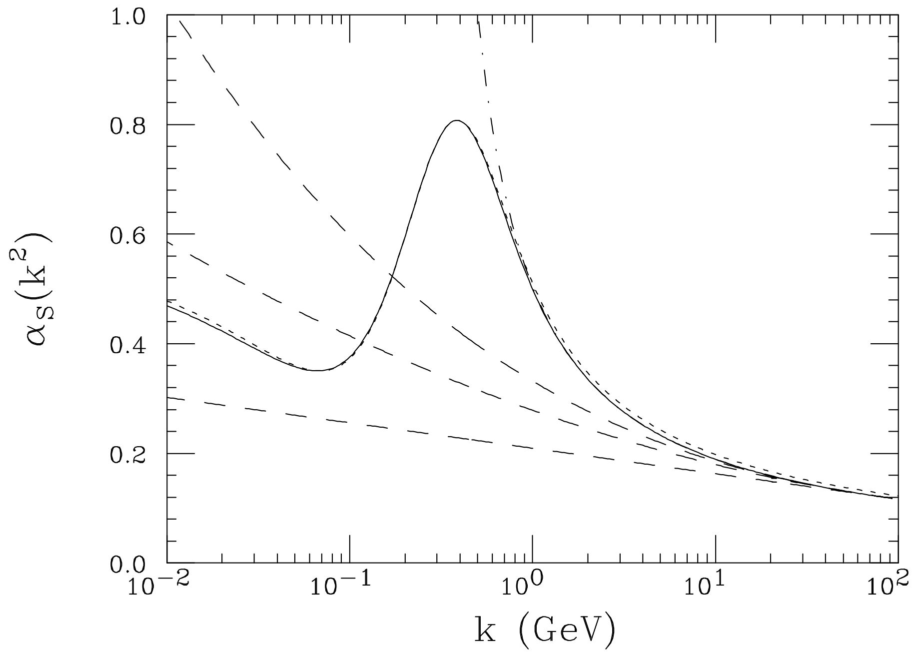

Figure 1: Model for the running coupling (solid curve), compared with one-loop perturbative form (dot-dashed) and expansions to first, second and third order in $\alpha_s(Q^2)$ (dashed, $Q = 91$ GeV). The dotted curve shows the 2-loop modification (2.3).

In order to relate the parameter Λ in eq. (1.1) to the scale parameter in some particular renormalization scheme, eq. (2.1) should be generalized to reproduce the 2-loop behaviour of $\alpha_s(k^2)$ at large k^2. As has been discussed for the simpler form (1.4) [8], this can be achieved without spoiling the analytic properties through a replacement of the form

$$x \to x\left[1+\left(\frac{\ln x}{2\pi}\right)^2\right]^b , \tag{2.3}$$

where

$$b = \frac{b_1}{2b_0^2} = \frac{3(153-19N_f)}{(33-2N_f)^2} . \tag{2.4}$$

As shown by the dotted curve in fig. 1, this makes only a very small difference at fixed values of the parameters, although of course these values would also need some readjustment. For simplicity we shall therefore use the 1-loop formula (2.2) in the remainder of this paper.

3. Power corrections

As a model for a QCD observable which is of perturbative order α_s^q and is expected to have a leading power correction of order $1/Q^p$, consider the integral [11]

$$F_{p,q}(Q) = \frac{p}{Q^p}\int_0^Q \frac{dk}{k} k^p \left[\alpha_s(k^2)\right]^q . \tag{3.1}$$

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Then to lowest order in $\alpha_s(Q^2)$ we have $F_{p,q} = F^{(0)}_{p,q}$ where

$$F^{(0)}_{p,q}(Q) = \Big[\alpha_s(Q^2)\Big]^q \ . \tag{3.2}$$

Using the explicit expression given by eqs. (1.1) and (2.2), we can evaluate the integral exactly, as shown for $p = q = 1$ by the solid curve in fig. 2. The difference between $F_{p,q}$ and $F^{(0)}_{p,q}$ arises from the running of $\alpha_s(k^2)$ in the integrand, which receives both perturbative and non-perturbative contributions. It is not possible to disentangle these unambiguously, because the two terms in eq. (2.2) separately give divergent contributions. A perturbative expansion in powers of $\alpha_s(Q^2)$ sees only the first term; expanding to $\mathcal{O}(\alpha_s^{q+N})$ means using

$$\Big[\alpha_s(k^2)\Big]^q_N \equiv \Big[\alpha_s(Q^2)\Big]^q \sum_{n=0}^{N} \frac{(n+q-1)!}{n!\,(q-1)!} \Big[b_0\alpha_s(Q^2)\ln\Big(Q^2/k^2\Big)\Big]^n \tag{3.3}$$

in eq. (3.1) to obtain

$$F^{(N)}_{p,q}(Q) = \Big[\alpha_s(Q^2)\Big]^q \sum_{n=0}^{N} \frac{(n+q-1)!}{(q-1)!} \left[\frac{2b_0}{p}\alpha_s(Q^2)\right]^n \ , \tag{3.4}$$

which diverges as $N \to \infty$. This is the infrared renormalon problem [11]–[16]. As one would expect from the corresponding curves in fig. 1, the perturbative estimate (3.4) of $F_{p,q}$ undershoots for low values of N, but becomes arbitrarily large for sufficiently high N (fig. 2).

To correct the $\mathcal{O}(\alpha_s^{q+N})$ perturbative estimate (3.4) we clearly have to write

$$F_{p,q}(Q) = F^{(N)}_{p,q}(Q) + \delta F^{(N)}_{p,q}(Q)\,, \tag{3.5}$$

where the correction term is

$$\delta F^{(N)}_{p,q}(Q) = \frac{p}{Q^p}\int_0^Q \frac{dk}{k} k^p \left(\Big[\alpha_s(k^2)\Big]^q - \Big[\alpha_s(k^2)\Big]^q_N\right) \ . \tag{3.6}$$

In the approximation scheme proposed in ref. [2], one notes that the two terms in the integrand of eq. (3.6) must become similar at values of k in the perturbative region, say $k > \mu_{\rm I}$, and that this region cannot contribute to the renormalon divergence. Therefore we neglect the integrand above $\mu_{\rm I}$, the 'infrared matching' scale, and write

$$\begin{aligned} \delta F^{(N)}_{p,q}(Q) &\simeq \frac{p}{Q^p}\int_0^{\mu_{\rm I}} dk\, k^{p-1} \left(\Big[\alpha_s(k^2)\Big]^q - \Big[\alpha_s(k^2)\Big]^q_N\right) \\ &= \left(\frac{\mu_{\rm I}}{Q}\right)^p F_{p,q}(\mu_{\rm I}) - G^{(N)}_{p,q}(\mu_{\rm I},Q)\,, \end{aligned} \tag{3.7}$$

where

$$G^{(N)}_{p,q}(\mu_{\rm I},Q) = \Big[\alpha_s(Q^2)\Big]^q \sum_{n=0}^{N} \frac{(n+q-1)!}{n!\,(q-1)!} \left[\frac{2b_0}{p}\alpha_s(Q^2)\right]^n \Gamma(n+1, p\ln(Q/\mu_{\rm I}))\ , \tag{3.8}$$

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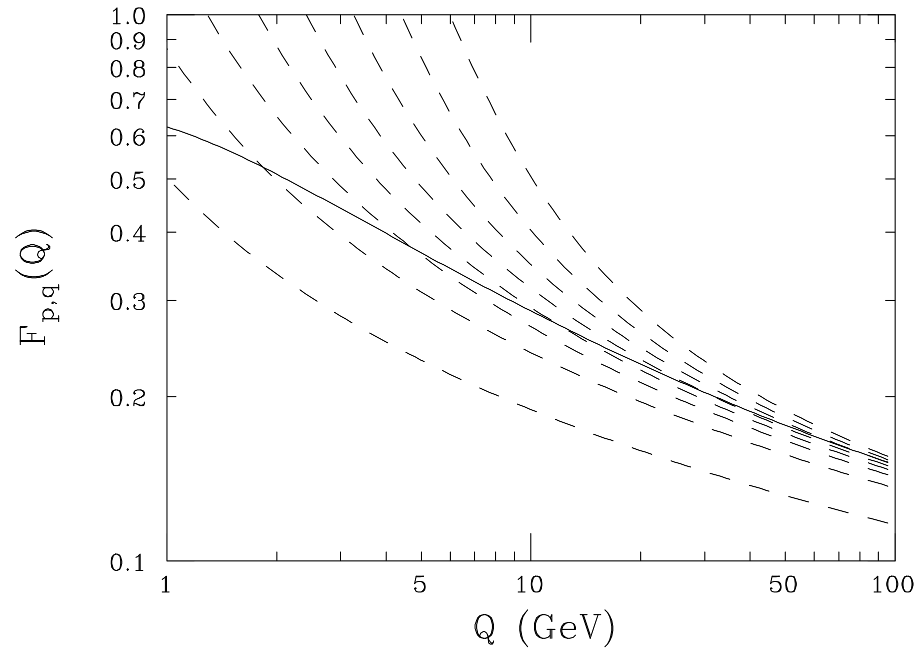

Figure 2: Exact value of the integral (3.1) for $p = q = 1$ (solid curve), compared with perturbative estimates (3.4) for $N = 0, \ldots, 7$ (dashed).

Γ being the incomplete gamma function,

$$\Gamma(n+1, z) = \int_z^\infty t^n \, e^{-t} \, dt \, . \tag{3.9}$$

The point of this exercise is that all the necessary non-perturbative information is now contained in the Q- and N-independent parameters $F_{p,q}(\mu_\mathrm{I})$, which represent weighted averages of α_s^q over the infrared region $0 < k < \mu_\mathrm{I}$. The resulting contribution to $F_{p,q}(Q)$ is a power correction of order $1/Q^p$. The second term on the right-hand side of eq. (3.7) subtracts the divergent renormalon part of the perturbative prediction. After this subtraction, the total correction $\delta F_{p,q}^{(N)}(Q)$ is not simply power-behaved, as may be seen from the difference between the exact result and any of the fixed-order curves in fig. 2.

For the model of the running coupling shown in fig. 1, we see that $\mu_\mathrm{I} = 2$ GeV is a reasonable matching scale, for which one finds the values of $F_{p,q}(\mu_\mathrm{I})$ given in table 1. The results $F_{1,1}(2\ \mathrm{GeV}) \simeq F_{2,1}(2\ \mathrm{GeV}) \simeq 0.5$ are consistent with the values deduced [1, 2, 3, 5, 14] from experimental data on event shapes [17, 18, 19] and structure functions [20, 21], respectively.[1]

q	$p=1$	$p=2$	$p=3$	$p=4$
1	0.511	0.450	0.410	0.388
2	0.283	0.218	0.176	0.155
3	0.170	0.114	0.081	0.064

Table 1: Values of $F_{p,q}(2\ \mathrm{GeV})$.

Using these values in eq. (3.7), we obtain the ($\mu_\mathrm{I} = 2$ GeV) 'matching estimates' of the integral (3.1), shown by the dot-dashed curves in fig. 3 for $N = 0, 1, 2, 6$. The $N = 1$ case corresponds to the most common situation, where we have to match to

[1]Note that in refs. [2, 3] the notation $F_{p,1}(\mu_\mathrm{I}) = \bar{\alpha}_{p-1}(\mu_\mathrm{I})$ was used.

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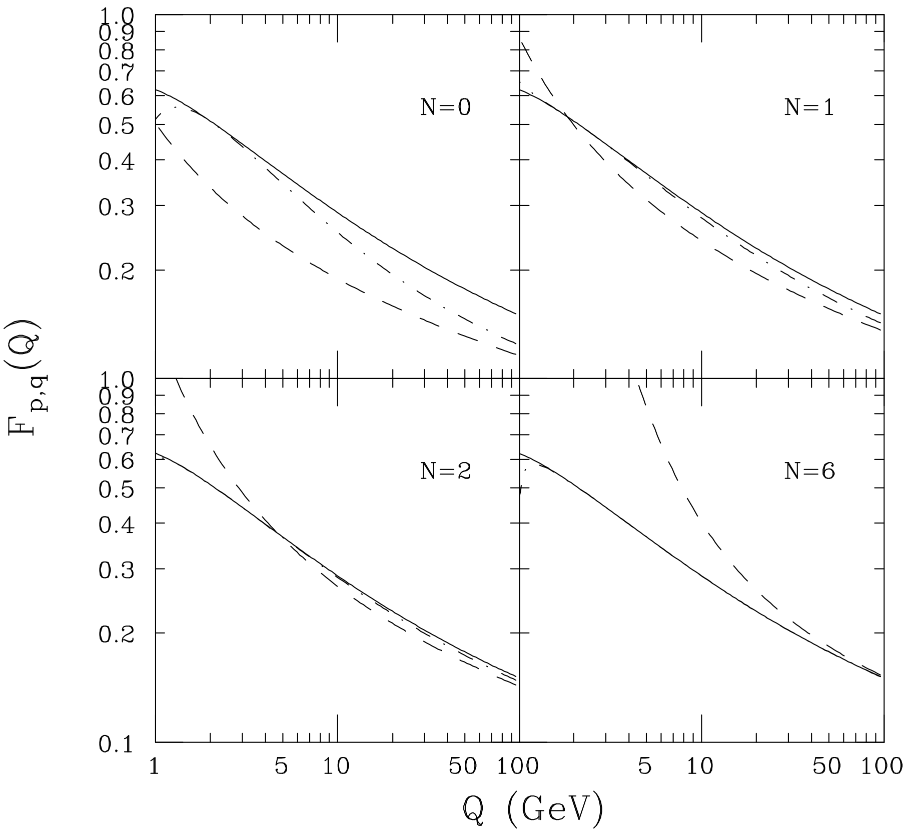

Figure 3: Exact value of the integral (3.1) for $p = q = 1$ (solid curves), compared with perturbative (dashed) and infrared matching (dot-dashed) estimates for $N = 0, 1, 2, 6$.

a NLO perturbative calculation. There is still some discrepancy at large Q because the matching in the interval $\mu_\text{I} < k < Q$ is not perfect. The discrepancy is reduced when the NNLO term is added ($N = 2$). The $N = 6$ case is shown to illustrate the cancellation of the renormalon and the improvement in the matching when very many perturbative terms are included.

Since we know the exact form of the integrand, we could of course improve the matching estimates at large Q by choosing a larger value of the matching scale μ_I. However in reality we usually know only the behaviour as $k^2/Q^2 \to 0$, and μ_I must be kept small in order for this behaviour to predominate in the correction term (3.6).

4. Renormalons

A common way to deal with the renormalon divergence of the perturbative estimate (3.4) as $N \to \infty$ is to Borel transform and define the sum by the principal value of the Borel integral [13, 16, 22, 23, 24, 25]. Recall that if

$$F(\alpha_s) = \sum_{n=0}^{\infty} c_n \alpha_s^{n+1} \tag{4.1}$$

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then the Borel transform is

$$\tilde{F}(u) = \sum_{n=0}^{\infty} \frac{c_n}{n!} u^n \tag{4.2}$$

and the sum would be defined as

$$F(\alpha_s) = P \int_0^\infty du\, \tilde{F}(u)\, e^{-u/\alpha_s}\,, \tag{4.3}$$

where P indicates the principal value. In the case of eq. (3.4) we have

$$\tilde{F}^{(\infty)}_{p,q}(u) = \frac{u^{q-1}}{(q-1)!}\,\frac{p}{p-2b_0 u} \tag{4.4}$$

and hence the 'renormalon estimate' of the perturbative part of $F_{p,q}(Q)$ is

$$F^{(ren)}_{p,q}(Q) = \left[\alpha_s(Q^2)\right]^q H_q\left(\frac{p}{2b_0\alpha_s(Q^2)}\right)\,, \tag{4.5}$$

where

$$H_q(z) = \frac{z^q}{(q-1)!}\left[e^{-z}\mathrm{Ei}(z) - \sum_{n=0}^{q-2} n!\, z^{-n-1}\right] \tag{4.6}$$

and Ei represents the exponential integral function,

$$\mathrm{Ei}(z) = -P\int_{-z}^{\infty} \frac{e^{-t}}{t}\,dt\,. \tag{4.7}$$

The non-perturbative part of $F_{p,q}(Q)$ then taken to be of the order of the residue of the renormalon pole at $u = p/2b_0$:

$$\delta F^{(ren)}_{p,q}(Q) = \left[\alpha_s(Q^2)\right]^q \delta H_q\left(\frac{p}{2b_0\alpha_s(Q^2)}\right)\,, \tag{4.8}$$

where

$$\delta H_q(z) = \frac{z^q}{(q-1)!}\,e^{-z}\,. \tag{4.9}$$

Taking $\pm\delta F^{(ren)}_{p,q}$ for the estimated non-perturbative part gives the upper/lower ends of the "error bars" shown in fig. 4.

The renormalon estimate (4.5) corresponds to a principal-value definition of the integral (3.1), i.e. to using the perturbative form (1.3) for $\alpha_s(k^2)$ everywhere and taking the average of contours running just above and below the point $k = \Lambda$. This implies that $\alpha_s(k^2) < 0$ for $k < \Lambda$, necessitating a positive non-perturbative contribution when $p = q = 1$ (see fig. 4). As p increases, the negative contribution from $k < \Lambda$ is suppressed and the region of positive α_s dominates, so that the a negative non-perturbative contribution is required. Nevertheless, for the simple model (2.2) assumed here for α_s, the renormalon estimate is quite satisfactory when $q = 1$, within the assumed uncertainty (4.8) shown by the error bars.

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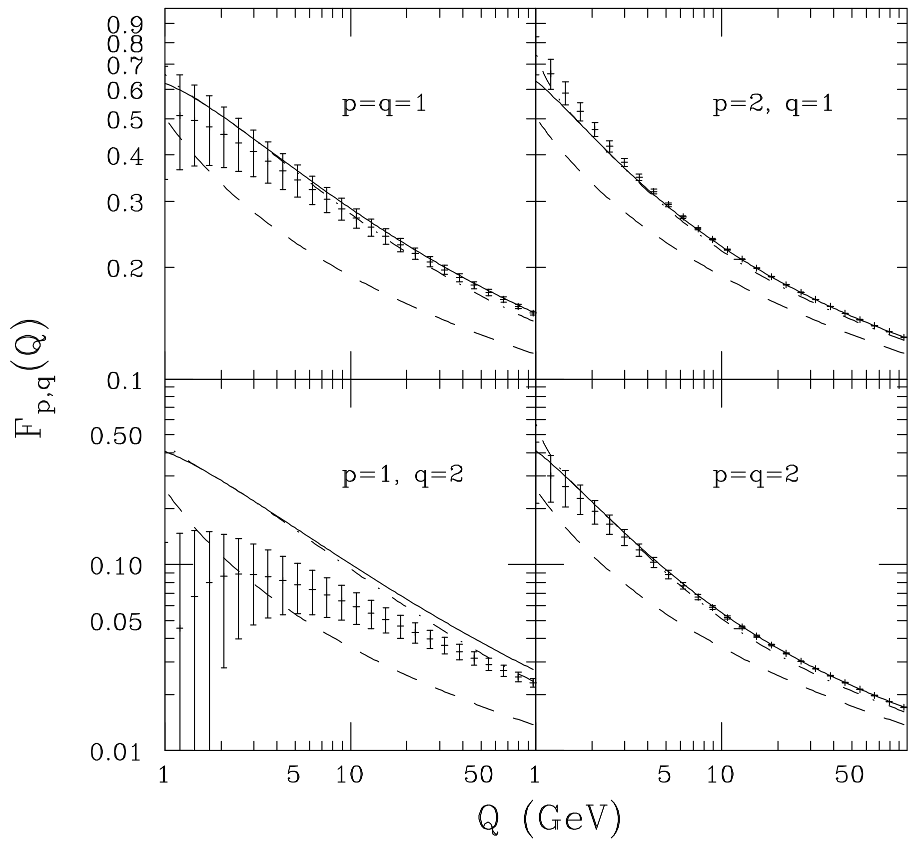

Figure 4: Exact values of the integral (3.1) for (solid curves), and renormalon estimates (points). For comparison the LO perturbative (dashed) and NLO matched (dot-dashed) estimates are also shown.

For higher powers of α_s, $q > 1$, the situation is different. Using the perturbative expression for $\alpha_s(k^2)$ in (3.1) means that the integrand has a multiple pole, and the physical interpretation of the principal value is not so obvious. A useful alternative interpretation (valid also for $q = 1$) is as follows. The function $F(Q) = F_{p,q}(Q)$ satisfies the differential equation

$$\frac{Q}{p}\frac{dF}{dQ} + F = \left[\alpha_s(k^2)\right]^q , \tag{4.10}$$

which has the general solution

$$F(Q) = \frac{p}{Q^p}\int_{Q_0}^{Q}\frac{dk}{k}k^p\left[\alpha_s(k^2)\right]^q , \tag{4.11}$$

where Q_0 is an arbitrary constant of integration. The renormalon estimate (4.5) corresponds to a particular choice of $Q_0 > 0$, namely the solution of

$$\alpha_s(Q_0^2) = \frac{p}{2b_0 z_q} , \tag{4.12}$$

$z = z_q$ being the point where $H_q(z) = 0$, i.e. where

$$\mathrm{Ei}(z) = e^z \sum_{n=0}^{q-2} n!\, z^{-n-1} . \tag{4.13}$$

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Explicit values are

$$z_1 = 0.373\,, \;\; z_2 = 1.347\,, \;\; z_3 = 2.342\,, \;\; z_q \sim q - 0.67\,. \tag{4.14}$$

Clearly the solution (4.11) corresponds (for $Q > Q_0$) to setting $\alpha_s(k^2) \equiv 0$ throughout $0 < k < Q_0$. Thus the renormalon estimate (4.5) is equivalent to using the perturbative expression for $\alpha_s(k^2)$ everywhere above an 'effective cutoff' $Q_0(p,q)$ given by eq. (4.12), and setting $\alpha_s(k^2) \equiv 0$ below that value. Using the 1-loop expression for $\alpha_s(Q_0^2)$, we obtain

q	$p = 1$	$p = 2$	$p = 3$	$p = 4$
1	1.45	1.20	1.13	1.10
2	3.85	1.96	1.57	1.40
3	10.40	3.22	2.18	1.80

Table 2: Values of $Q_0(p,q)/\Lambda$.

$$Q_0(p,q) = \Lambda\, e^{z_q/p} \sim \Lambda\, e^{(q-0.67)/p}\,. \tag{4.15}$$

Note that the effective cutoff $Q_0(p,q)$ grows rapidly with increasing q, especially for $p = 1$ (table 2).

Setting $\alpha_s(k^2) = 0$ for $k < Q_0(p,q)$ is equivalent to the principal-value definition of the integral in (3.1), because the principal value of the integral from 0 to $Q_0(p,q)$ is zero. As q increases, the higher effective cutoff leads to a lower renormalon estimate of the integral, necessitating a non-perturbative correction greater than the estimate (4.8) (see fig. 4). The fact that the cutoff on α_s is imposed at a scale that depends on p and q makes the renormalon approach look somewhat unnatural. Given that the analyticity properties are destroyed by the principal value prescription, one might as well choose to set $\alpha_s(k^2) = 0$ below some fixed value of Q_0, to be treated as a free parameter.

Another way of defining the divergent sum (3.4) is by truncating the series at its smallest term [13], i.e. summing up to the integer nearest to

$$N(Q) = \frac{p}{2b_0\alpha_s(Q^2)} - q + \tfrac{1}{2} \tag{4.16}$$

and assigning an uncertainty equal to the smallest term. This is valid if the series is an asymptotic expansion. The resulting 'optimal truncation' estimate is again a solution of the differential equation (4.10), but the effective cutoff $Q_0(p,q)$ is now given by $N(Q_0) = 0$, i.e.

$$\alpha_s(Q_0^2) = \frac{p}{2b_0(q - \frac{1}{2})} \tag{4.17}$$

which is just eq. (4.12) with z_q replaced by $q - \frac{1}{2}$. In view of eqs. (4.14), one then expects results very similar to those of the renormalon approach, which is indeed the case (fig. 5).

5. Conclusions

The simple model of the running coupling introduced in sect. 2 has allowed us to study various aspects of the estimation of QCD power corrections in some detail. For this purpose we have used the toy model observable $F_{p,q}(Q)$ defined in eq. (3.1).

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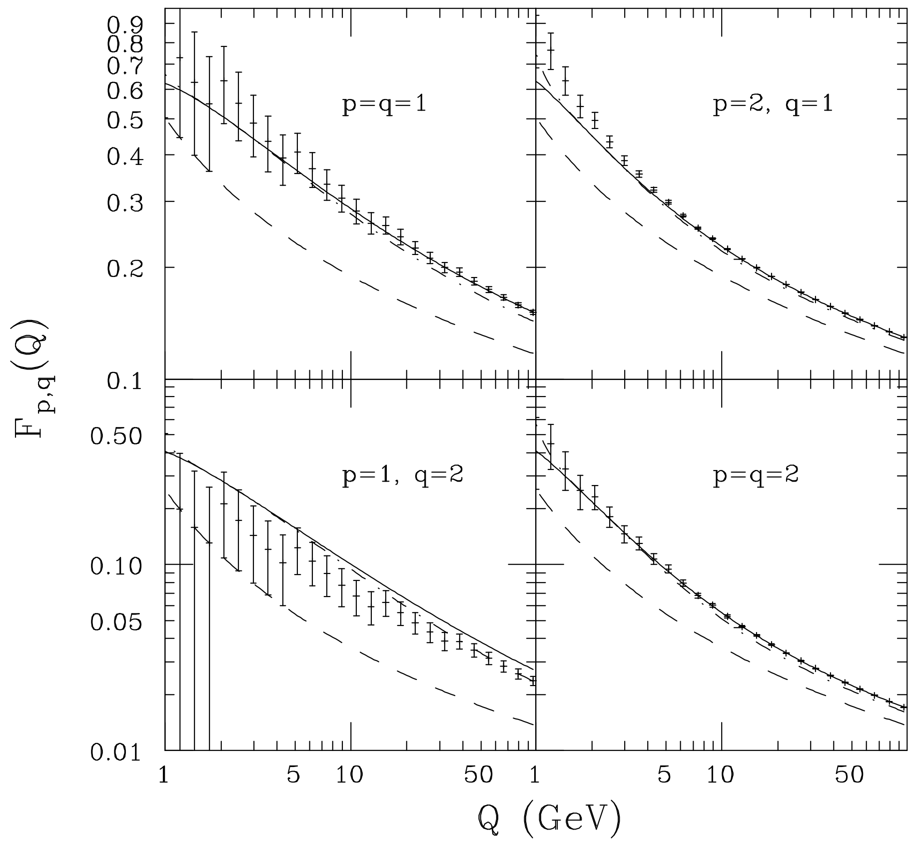

Figure 5: Exact values of the integral (3.1) (solid curves), and optimally truncated estimates (points). For comparison the LO perturbative (dashed) and NLO matched (dot-dashed) estimates are also shown.

The 'infrared matching' procedure introduced in ref. [2] gives fairly good results when matched with the NLO ($N = 1$) perturbative estimate, improving rapidly as higher orders are included (fig. 3). In principle it eliminates the renormalon problem completely, at the price of introducing new non-perturbative parameters, which are the moments of α_s at low scales (table 1). The simple model expresses these moments in terms of its own non-perturbative parameters, b, c and p in eq. (2.1). For the present study the latter were fixed at the values in eq. (2.2), which give agreement with a range of data on power corrections. In future these parameters could be varied to achieve a best fit. For serious comparisons with data, the 2-loop modification (2.3) should be used, and the varying number of active flavours would also need to be taken into account.

A renormalon analysis of the perturbation series suggests another way to estimate power corrections, either through Borel transformation or by optimal truncation of the series. Although this approach seems to involve no non-perturbative parameters, it is equivalent to using the perturbative expression for the running coupling above a cutoff Q_0 and setting it to zero below Q_0. Since the value of the effective cutoff Q_0 depends on the details of the integrand, viz. the powers of momentum and α_s (table 2), this

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procedure cannot correspond to any universal model for α_s. In particular, the effective cutoff increases rapidly with the power of α_s, and therefore renormalon estimates of higher-order effects will fall below those of any universal model for sufficiently high orders.

Whether a universal model of the running coupling, including non-perturbative terms, can be extended to higher orders is of course open to question anyway. For the parameter values used in eq. (2.2), the model coupling is nowhere larger than 0.8 (fig. 1), and so an expansion in powers of $\alpha_s(k^2)/\pi$ does not seem unreasonable. The hope is also that large coefficients due to infrared renormalons should not appear in this expansion.

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